



## Corrections to “Lower Bounds on Q for Finite Size Antennas of Arbitrary Shape”

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# Corrections to “Lower Bounds on Q for Finite Size Antennas of Arbitrary Shape”

Oleksiy S. Kim

Equations (24) and (25) in Appendix B of [1] should respectively read as

$$\begin{aligned} \int_{V_\infty} -(\nabla G_1)G_2^* - \hat{\mathbf{r}}jk \frac{e^{jk(\mathbf{r}_1-\mathbf{r}_2)\cdot\hat{\mathbf{r}}}}{16\pi^2|\mathbf{r}|^2} dV &= -\frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|} \frac{\cos(k|\mathbf{r}_{12}|)}{8\pi} \\ &-j \frac{2\mathbf{r}_1}{8\pi k^2} \left( \frac{\sin(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^3} - \frac{k \cos(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^2} \right) \\ &-j \frac{|\mathbf{r}_1|^2 - |\mathbf{r}_2|^2}{8\pi k^2} \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|^2} \left( \frac{k^2 \sin(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|} \right. \\ &\left. - 3 \left( \frac{\sin(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^3} - \frac{k \cos(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^2} \right) \right) \end{aligned} \quad (1)$$

and

$$\begin{aligned} \int_{V_\infty} j(\nabla G_1)G_2^* - \hat{\mathbf{r}}k \frac{e^{jk(\mathbf{r}_1-\mathbf{r}_2)\cdot\hat{\mathbf{r}}}}{16\pi^2|\mathbf{r}|^2} dV &= j \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|} \frac{\cos(k|\mathbf{r}_{12}|)}{8\pi} \\ &- \frac{\mathbf{r}_{12}}{8\pi k^2} \left( \frac{\sin(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^3} - \frac{k \cos(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^2} \right) \\ &- \frac{\mathbf{r}_1 + \mathbf{r}_2}{8\pi k^2} \left( \frac{\sin(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^3} - \frac{k \cos(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^2} \right) \\ &- \frac{|\mathbf{r}_1|^2 - |\mathbf{r}_2|^2}{8\pi k^2} \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|^2} \left( \frac{k^2 \sin(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|} \right. \\ &\left. - 3 \left( \frac{\sin(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^3} - \frac{k \cos(k|\mathbf{r}_{12}|)}{|\mathbf{r}_{12}|^2} \right) \right) \quad (1a) \\ &= j \frac{\mathbf{r}_{12}}{2} \text{Re}\{G_{12}\} - \frac{1}{2k^2} \text{Im}\{\nabla_1 G_{12}\} \\ &- \frac{\mathbf{r}_1 + \mathbf{r}_2}{2k^2} \text{Im}\{\nabla_1 G_{12} \cdot \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|^2}\} \\ &+ \frac{|\mathbf{r}_1|^2 - |\mathbf{r}_2|^2}{2k^2 |\mathbf{r}_{12}|^2} \text{Im}\{\mathbf{r}_{12} k^2 G_{12} + 3\nabla_1 G_{12}\}. \quad (1b) \end{aligned}$$

All other results in [1] do not involve the coordinate-dependent terms (those with  $\mathbf{r}_1 + \mathbf{r}_2$  and  $|\mathbf{r}_1|^2 - |\mathbf{r}_2|^2$  multipliers), in which the error actually occurs, and thus, are not affected. The contribution of the coordinate-dependent terms is insignificant for  $ka < 0.5$ , whereas for larger  $ka$ , where the contribution gradually increases, the  $Q$  itself becomes too low to be reliably related to the bandwidth.

Further numerical results exemplifying and substantiating the general applicability of a procedure for determining the lower bound on  $Q$  outlined in Section V in [1] can be found in [2].

The expressions for the stored energies and the radiated power of arbitrary electric and magnetic currents presented in [1] (Tables I and II) can also be used for computing the  $Q$  of electrically small antennas loaded with magneto-dielectric materials, as demonstrated in [3] in the context of a surface integral equation method.

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## REFERENCES

- [1] O. S. Kim, “Lower bounds on Q for finite size antennas of arbitrary shape,” *IEEE Trans. Antennas Propagat.*, vol. 64, no. 1, pp. 146–154, January 2016.
- [2] —, “Lower bounds on Q of some dipole shapes,” in *Proc. IEEE Antennas Propag. Soc. Int. Symp.*, Fajardo, Puerto Rico, 26 June - 1 July 2016, pp. 415–416.
- [3] —, “Computation of the radiation Q of dielectric-loaded electrically small antennas in integral equation formulations,” in *Proc. IEEE/ACES Int. Conf. Wireless Information Technology and Systems (ICWITS) and Applied Computational Electromagnetics (ACES)*, Honolulu, Hawaii, USA, 13-17 March 2016.